

Forming and Solving Optimization Problems Theory and Software Tools

Imre Pólik, PhD

McMaster University
School of Computational Engineering and Science

February 14, 2008

Outline

Outline

Optimization
problems

Modelling

Solvers

Modelling
environments

Resources

- 1 Optimization problems
- 2 Modelling
- 3 Solvers
- 4 Modelling environments

Optimization problems

Outline

Optimization
problems

Modelling
Solution
Problem input/output

Modelling

Solvers

Modelling
environments

Resources

- Linear (LP)
- Convex quadratic (QP)
- Piecewise linear, quadratic
- Second order (SOCP)
- Semidefinite (SDP)
- Convex nonlinear (NLP)
- Mixed integer, binary (MILP, MIQP)
- Nonconvex quadratic
- Quadratically constrained quadratic (QCQP)
- Geometric (GP)
- Nonlinear semidefinite (NLSDP)
- General nonlinear (NLP)
- Global optimization

Choosing a solver

- Identify the problem type
- Decision Tree for Optimization Software
- Requirements
 - accuracy
 - solution time
 - problem size
 - platform, programming environment
 - licensing, cost
- Choose the most specific solver
- Reformulate the problem to use a better solver
- Starting solution

Entering the problem

- Native solver input
 - usually text-based
 - solver specific
- Modelling environment
 - standard interface to many solvers
 - custom GUI design
 - automatic derivatives
 - plotting
 - (Matlab), AMPL, GAMS, MPL, ILOG, XPRESS-IVE, AIMMS, LINDO, CVX, YALMIP
- Spreadsheet application
- Database integration
- Callable library, embedded solvers
 - tighter integration
 - C, C++, Fortran, Java, .NET, C#, VB, Delphi, Python, (Matlab, R, etc.)

Interpreting the solution

- Solver exit flag
 - (nearly) optimal
 - infeasible problem
 - numerical problems
 - lack of progress
- Does it make sense?
 - better than previous solution
 - too good to be true
 - missing constraints, incorrect data
- Optimality
 - the solution is rarely optimal
 - the role of convexity
- Infeasible problem
 - certificate
 - conflicting constraints

Outline

Optimization
problems

Modelling

Reformulating the
problem
Managing constraints

Solvers

Modelling
environments

Resources

Solvers

- 1 Optimization problems
- 2 Modelling
 - Reformulating the problem
 - Managing constraints
- 3 Solvers
- 4 Modelling environments

Equalities and inequalities

- Equalities into inequalities

$$h(x) = 0 \Leftrightarrow h(x) \leq 0, h(x) \geq 0$$

- possibly empty interior
- increased problem size

$$h(x) = 0 \Leftrightarrow h(x) \leq \varepsilon, h(x) \geq -\varepsilon$$

- nonempty interior
- explicit tolerance

- Inequalities into equalities

$$g(x) \leq 0 \Leftrightarrow g(x) + s = 0, s \geq 0$$

- slack variable
- increased problem size

$$g(x) \leq 0 \Leftrightarrow g(x) = -z^2$$

- nonlinearity

Managing constraints and variables

- New variables
 - trigonometry

$$u, \pm\sqrt{1-u^2} \Leftrightarrow \sin \alpha, \cos \alpha$$

- grouping common terms
- Smart variables
 - extremely important in integer programming
- Adding constraints gradually
 - only a few constraints are active at optimality
 - column generation
 - quick resolve

Outline

Optimization
problems

Modelling

Reformulating the
problem
Managing constraints

Solvers

Modelling
environments

Resources

Relaxation

- Remove some constraints
 - integrality
 - semidefiniteness
- Solve the easier problem
 - LP instead of MILP
 - SDP instead of NLSDP
- Hope for the best
 - manage unsatisfied constraints
 - provides a bound on the optimum
 - branch-and-bound
- Powerful technique

Outline

Optimization
problems

Modelling

Solvers

Linear/quadratic
optimization

Conic optimization

Nonlinear
optimization
NEOS

Modelling
environments

Resources

Solvers

- 1 Optimization problems
- 2 Modelling
- 3 Solvers
 - Linear/quadratic optimization
 - Conic optimization
 - Nonlinear optimization
 - NEOS
- 4 Modelling environments

LP/MILP

- CPLEX, XPRESS-MP
 - LP, MIP, QP, MIQP
 - model development environment
 - callable from C, C++, Java, Fortran, VB6 and .NET
 - industry leaders
 - free limited student version
- LpSolve, Soplex, CLP
 - LP/MILP
 - free, open source

Outline

Optimization
problems

Modelling

Solvers

Linear/quadratic
optimization

Conic optimization

Nonlinear
optimization
NEOS

Modelling
environments

Resources

Conic optimization

- LP, SOCP, SDP
- CSDP, SDPA
 - written in C
 - large scale problems
 - parallel, distributed operation
 - no SOCP
- SeDuMi, SDPT3
 - written in Matlab
 - mixed LP/SOCP/SDP
 - easy to use
- Others: DSDP, SDPLR, PENNON, SBMethod,

Nonlinear optimization

- IPOPT
 - open source
- SNOPT, KNITRO, MINOS, etc.
 - commercial
- LGO, Baron, Globsol
 - global optimization
 - reasonable sizes
- Issues
 - availability of gradient, Hessian
 - cost of function evaluation
 - starting point

Outline

Optimization
problems

Modelling

Solvers

Linear/quadratic
optimization

Conic optimization

Nonlinear
optimization

NEOS

Modelling
environments

Resources

NEOS

- Free online access to solvers
- Various interfaces
 - web
 - email
 - modelling languages
- Great for comparisons, testing

Outline

Optimization
problems

Modelling

Solvers

Linear/quadratic
optimization

Conic optimization

Nonlinear
optimization

NEOS

Modelling
environments

Resources

Outline

Optimization
problems

Modelling

Solvers

**Modelling
environments**ILOG OPL
Development Studio

XPRESS-IVE

AIMMS

AMPL and GAMS

Matlab

CVX

YALMIP

Resources

Modelling environments

- 1 Optimization problems
- 2 Modelling
- 3 Solvers
- 4 Modelling environments
 - ILOG OPL Development Studio
 - XPRESS-IVE
 - AIMMS
 - AMPL and GAMS
 - Matlab
 - CVX
 - YALMIP

Outline

Optimization
problems

Modelling

Solvers

Modelling
environmentsILOG OPL
Development Studio
XPRESS-IVEAIMMS
AMPL and GAMS

Matlab

CVX

YALMIP

Resources

ILOG OPL Development Studio

- Built around CPLEX
- LP, QP, MILP, MIQP
- Extensive GUI creation
- Mapping tools
- Integration with C++, Java
- Spreadsheet and database integration
- Embeddability
- ILOG Optimization Decision Manager
 - planning scheduling

XPRESS-IVE

Outline

Optimization
problems

Modelling

Solvers

Modelling
environments

ILOG OPL
Development Studio

XPRESS-IVE

AIMMS

AMPL and GAMS

Matlab

CVX

YALMIP

Resources

- Built around the XPRESS-MP solvers
- LP, QP, MILP, MIQP
- Integration with C++, Java
- Spreadsheet and database integration
- Embeddability
- Abilities similar to ILOG OPL

AIMMS

Outline

Optimization
problems

Modelling

Solvers

Modelling
environments

ILOG OPL
Development Studio
XPRESS-IVE

AIMMS

AMPL and GAMS

Matlab

CVX

YALMIP

Resources

- General nonlinear modelling environment
- A handful of solvers (LP, MILP, QP, MIQP, NLP)
- Graphical user interface
- GUI building
- Database, spreadsheet integration
- Free student version

AMPL and GAMS

- General nonlinear modelling environments
- Plenty of solvers
- Free student version available
- Text-based, no GUI (AMPL has a simple one)
- No SDP

Outline

Optimization
problems

Modelling

Solvers

Modelling
environments

ILOG OPL
Development Studio

XPRESS-IVE

AIMMS

AMPL and GAMS

Matlab

CVX

YALMIP

Resources

Matlab

- Powerful computational background
- Optimization Toolbox (LP, QP, NLP)
- Robust Control Toolbox (SDP)
- Not embeddable

Outline

Optimization
problems

Modelling

Solvers

Modelling
environments

ILOG OPL
Development Studio

XPRESS-IVE

AIMMS

AMPL and GAMS

Matlab

CVX

YALMIP

Resources

Outline

Optimization
problems

Modelling

Solvers

Modelling
environmentsILOG OPL
Development Studio

XPRESS-IVE

AIMMS

AMPL and GAMS

Matlab

CVX

YALMIP

Resources

- Disciplined convex optimization
- Maintain convexity by construction
- Only two solvers (SeDuMi and SDPT3)
- LP, QP, SOCP, SDP, GP
- Written in Matlab

YALMIP

- Written in Matlab
- Free, open source
- Powerful modelling abilities
- Covers all kinds of optimization problems
- Connected to more than 30 solvers

Outline

Optimization
problems

Modelling

Solvers

Modelling
environments

ILOG OPL
Development Studio

XPRESS-IVE

AIMMS

AMPL and GAMS

Matlab

CVX

YALMIP

Resources

Resources

Outline

Optimization
problems

Modelling

Solvers

Modelling
environments

Resources

- NETLIB
 - Software repository
 - benchmark problems
- COIN-OR
 - Open source initiative for optimization
 - LP, MILP, QP, MIQP, SDP, NLP
 - automatic differentiation
- Decision Tree for Optimization Software
- Hans Mittelmann's benchmarks