New features and improvements in SeDuMi

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Outline

What is SeDuMi?

- Optimization over symmetric cones
  - linear
  - second order
  - semidefinite
  - complex variables
  - free variables
- Interior point method
  - primal-dual
  - self dual embedding
  - predictor-corrector scheme
- Open source
  - GPL
  - Matlab, C
- Widely used
  - both industry and academics

Problem definition

- Primal-dual conic optimization

$$\begin{aligned} \min\ & c^T x & \qquad \max\ & b^T y \\ & Ax = b & & A^T y + s = c \\ & x \in K & & s \in K, \end{aligned}$$

  where K is

linear	$\mathbb{R}^k_+$
second order	$\left\{x \in \mathbb{R}^\ell : x_1 \geq \|x_{2:\ell}\|\right\}$
semidefinite	$\left\{X \in \mathbb{R}^{m\times m} : X \succeq 0\right\}$

  or any product of these.

- User input≠solver input
  - free variables
  - rotated Lorentz cones
  - complex data

History

late 1997	Jos F. Sturm starts SeDuMi
summer 1998	SeDuMi 1.0
November 2002	SeDuMi 1.05R5 (last version)
November 2003	Jos dies
October 2004	AdvOL at McMaster takes over
June 2005	SeDuMi 1.1 (new version)

Our approach

- Legal issues
- New website
  - `http://sedumi.mcmaster.ca`
  - powered by Mambo Open Source
  - 65000 visits
  - 1300 registered users
  - user forum
- Bottlenecks
  - memory usage
  - time per iteration
  - Matlab
- Strengths
  - robustness
  - accuracy
- Trends
  - BLAS/LAPACK
  - 64bit
  - parallelization
- User input

Development plans I.

- Preprocessing
  - widespread use in LP codes
  - significant savings
  - very little for SOCP/SDP
  - decomposition (Kojima et. al, Plaza Martínez/Krishnan, Young/Anjos)
  - mixed techniques
- Adaptive techniques
  - before and during the algorithm
  - corrector type
  - update method
  - step-differentiation
  - neighbourhood parameters
  - starting point (Freund)
- Sparse/dense issues
  - currently inefficient
  - BLAS/LAPACK (also in parallel)
  - ScaLAPACK (distributed)

Development plans II.

- Converters
  - SDPA↔SeDuMi (SDP)
  - Mosek↔SeDuMi (SOCP)
  - modelling language support (YALMIP)
- Miscellaneous
  - improved infeasibility detection
  - QP as SOCP
  - new search directions
  - new scaling techniques
- Implementational issues
  - 64bit
  - parallelization
- Platform independent installer
- New user guide

Preprocessing I.

- Detect diagonal SDP blocks

$$\mathrm{Diag}\,(x) \succeq 0 \Leftrightarrow x \geq 0$$

  - LP in SDP
  - source: user or other format
- Free variables
  - Splitting: $x = x_+ - x_-$, $x_+, x_- \geq 0$
  - SOCP: $x_0 \geq \|x\|$
- Detecting split free variables
  - improved robustness, accuracy, less memory
  - nonlinearity, fill-in

Preprocessing II.

Kocvara problems	Time (s)								Iter		Time/iter (s)	
	Total		Pre		IPM		Post					
buck1	0.3	0.2	0.0	0.0	0.3	0.2	0.0	0.0	22	22	0.0	0.0
buck2	12.7	5.8	0.0	0.0	12.7	5.8	0.0	0.0	38	40	0.3	0.1
buck3	1152.5	427.5	0.7	0.3	1150.8	426.9	1.0	0.3	64	75	18.0	5.7
buck4	12329.8	3971.4	6.3	2.1	12314.5	3966.6	9.0	2.7	64	76	192.4	52.2
buck5	m	122176.5	m	29.3	m	122105.9	m	41.3	m	110	m	1110.1
mater-1	0.3	0.2	0.0	0.0	0.3	0.2	0.0	0.0	21	21	0.0	0.0
mater-2	1.2	1.2	0.0	0.1	1.2	1.1	0.0	0.0	25	25	0.0	0.0
mater-3	5.6	5.6	0.1	0.9	5.5	4.7	0.0	0.0	27	27	0.2	0.2
mater-4	23.9	30.9	0.3	10.6	23.5	20.3	0.1	0.1	30	30	0.8	0.7
mater-5	69.7	108.1	0.8	50.2	68.7	57.7	0.3	0.2	33	33	2.1	1.7
mater-6	184.4	420.8	2.1	216.1	181.7	204.4	0.7	0.3	35	36	5.2	5.7
shmup1	0.4	0.3	0.0	0.0	0.3	0.3	0.0	0.0	17	16	0.0	0.0
shmup2	741.2	512.6	0.6	0.5	739.9	511.5	0.7	0.6	39	42	19.0	12.2
shmup3	8484.0	7061.4	5.2	4.1	8473.0	7053.0	5.8	4.2	51	61	166.1	115.6
shmup4	68360.6	61161.8	32.5	24.0	68292.9	61112.6	35.3	25.1	63	88	1084.0	694.5
trto1	0.2	0.1	0.0	0.0	0.2	0.1	0.0	0.0	21	21	0.0	0.0
trto2	14.7	4.2	0.0	0.0	14.6	4.2	0.0	0.0	36	37	0.4	0.1
trto3	752.4	128.8	0.6	0.2	750.9	128.6	0.8	0.1	50	59	15.0	2.2
trto4	9080.1	2225.4	5.6	1.2	9066.7	2222.8	7.8	1.4	61	73	148.6	30.4
trto5	m	45465.7	m	16.0	m	45428.3	m	21.3	m	95	m	478.2
vibra1	0.3	0.2	0.0	0.0	0.3	0.2	0.0	0.0	26	25	0.0	0.0
vibra2	12.5	5.4	0.0	0.0	12.4	5.3	0.0	0.0	41	43	0.3	0.1
vibra3	1104.5	301.2	0.7	0.2	1102.9	300.6	1.0	0.3	68	69	16.2	4.4
vibra4	13565.5	3812.2	6.3	2.0	13550.1	3807.4	9.0	2.7	77	78	176.0	48.8
vibra5	m	123712.3	m	32.0	m	123639.0	m	41.3	m	126	m	981.3

Athlon64 3500+, 2GB RAM, SuSE 9.3, Matlab 7.0 (64bit)

Parallelization

- BLAS/LAPACK
  - AMD, Intel, ATLAS or through Matlab
  - matrix products/factorizations
  - huge speedup
  - automatic parallelization
- OpenMP
  - selected compilers (Pathscale, IBM, Intel)
  - forming ADA^T
  - only $k^{0.6}$ speedup

Step differentiation

- Different primal-dual stepsizes
  - Faster progress: less iterations
  - Numerical stability
- But: not too aggressively
- Turned on
  - after 15 iterations, or
  - if CG refinement is used, or
  - if we are close to convergence.
- It works!

Future plans and work in progress

- Preprocessing
  - finding block-diagonal structure
  - decomposing narrow-band matrices
  - LP techniques on the linear part
  - developing new techniques
- Adaptive techniques
  - gathering data
- Infeasibility detection
  - infeasible SDP library
  - weak infeasibility
- Linear algebra
  - sparse BLAS
  - ScaLAPACK

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