CES 703 Assignment #3 Due date: Friday, February 22, 4:00pm

Dr. Imre Pólik (40+20 points)

Electronic submission is preferred. Do not print anything that's generated electronically. Whatever can be submitted electronically, please do it that way. Please submit the source code and the output along with the solutions.

For the programming problems you can use your favourite language (C, Fortran, Matlab, Python, Java, etc.) provided that it can handle the problem. If you choose a compiled language please provide instructions for compilation (possibly a Makefile).

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1. (20 points) Consider the following matrix:

$$\mathcal{A} \in \mathbb{R}^{n^2 \times n^2}$$
$$\mathcal{A}\mathrm{vec}(X) = \mathrm{vec}(UXU),$$

   where U and X are $n \times n$ symmetric matrices, and $\mathrm{vec}(X)$ is the vectorization of X, i.e., it contains the columns of X stacked together in one long column vector. Matrix $\mathcal{A}$ maps an $n \times n$ matrix to another one.

   (a) Form $\mathcal{A}$ explicitly in terms of U. What is the structure of this matrix? (Hint: $\otimes$)

   (b) Analyze the applicability of direct methods to find the eigenvalues of $\mathcal{A}$. Do they work here?

   (c) Now consider iterative methods. Choose a suitable algorithm to find the eigenvalues of $\mathcal{A}$. Try it for some simple cases when you know the eigenvalues of U. What do you observe?

   (d) After all, what is the best way to find the eigenvalues of this matrix? Can you get them directly from U?

2. (20 points) Implement the bisection algorithm to find the eigenvalues of a matrix in an interval $[\alpha, \beta]$. You can use the pseudocode on page 229 in Demmel's book. Experiment with different strategies to choose the bisecting point: midpoint, golden section, random, two bisecting points at the thirds, etc.

Bonus: (20 points) Find an efficient algorithm to compute the QR decomposition of a matrix $A = R + uv^T$, where R is upper triangular, and u and v are column vectors. (Hint: Use Givens rotations. The complexity of your algorithm should be $\mathcal{O}(n^2)$ instead of $\mathcal{O}(n^3)$.) [From James W. Demmel. Applied Numerical Linear Algebra. SIAM, Philadelphia, 1997.]