MORE WAYS TO USE DUAL INFORMATION IN MILP

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AGENDA

1. Reusing Dual Information
2. Dual Tightening
3. Branching Variable Selection
4. Computational Results

DUAL INFORMATION IN BRANCH AND BOUND

- Dual information:
  - Dual solutions from feasible node LPs
  - Dual rays from infeasible node LPs
- Tree search generates a large amount of dual information
- Globally valid and can be reused elsewhere in tree
- How?

REUSING DUAL INFORMATION

DUAL INFORMATION IN BRANCH AND BOUND

- Traditional usage
  - Reduced cost fixing (local and root)
  - Conflict analysis
- New ways
  - Keep dual rays and solutions
  - Prune nodes
  - Subtree bounds, simulate subtrees (lookahead branching)
  - Symmetry
  - Dual ray fixing (no incumbent or bound needed)
  - Dual tightening (reduced cost fixing)
  - Variable selection

- Node LP problem

$$\begin{aligned} z^* = \min\ & cx \\ \text{s.t. } & L \leq Ax \leq U \\ & \ell \leq x \leq u \end{aligned}$$

- For a dual solution y, dual objective value is

$$z(y) = \sum_{i:y_i>0} L_i y_i + \sum_{i:y_i<0} U_i y_i + \sum_{j:r_j(y)>0} \ell_j r_j(y) + \sum_{j:r_j(y)<0} u_j r_j(y)$$

  where $r_j(y) = c_j - A_j^T y$ is the reduced cost of variable j.
- From LP duality: $z^* \geq z(y)$.

- We can tighten bounds by LP duality
  - Let $\bar{z}$ be the current cutoff/upper bound and $r_j(y) > 0$
  - If we fix $l_j = u_j$, then the resulting dual objective value is

$$z'(y) = z(y) + (u_j - l_j) r_j(y)$$

  - We can tighten u_j, if $z'(y) > \bar{z}$
- Tightening function for variable j

$$\pi_j(y) = \frac{\bar{z} - z(y)}{r_j(y)}$$

- We can tighten u_j if

$$\min_{y: r_j(y) > 0} \pi_j(y) < u_j - l_j$$

- Same applies to l_j

TIGHTENING FUNCTION - PROPERTIES

$$\pi_j(y) = \frac{\bar{z} - z(y)}{r_j(y)}$$

- Numerator is piecewise linear, convex
- Denominator is linear
- The fraction is piecewise pseudolinear, overall quasiconvex
- Due to pseudolinearity the minimum is taken at the boundaries of the regions in the numerator
  - That is where the reduced costs change sign
- Can be minimized with a modified simplex algorithm, but expensive

TIGHTENING FUNCTION

$$\pi_j(y) = \frac{\bar{z} - z(y)}{r_j(y)}$$

Classical approach:

- Minimize numerator
  - Solve node LP and use the optimal dual solution y^*
- Maximize denominator (keeping LP optimal): find true shadow prices
  - Sensitivity analysis, expensive

New approaches:

1. Use dual solutions collected from other node LPs
2. Use dual solutions collected from dual simplex iterations
3. Search neighborhood of y^*
4. Search an improving direction for y^*

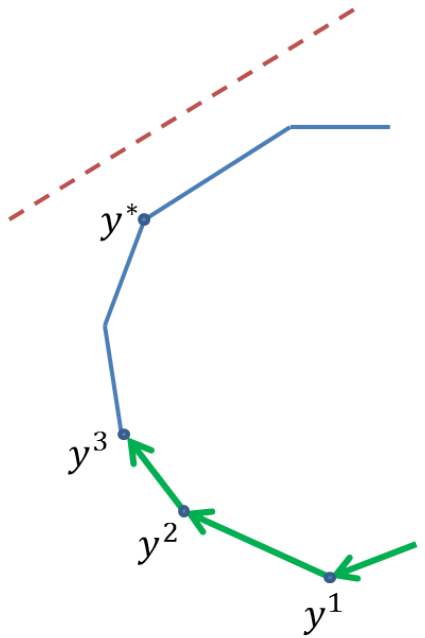
1. TREE/SIMPLEX DUAL SOLUTIONS

Dual solutions from tree search

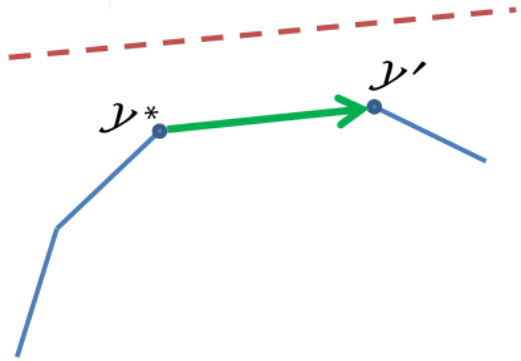
- A dynamic list of dual solutions as they are found in tree
- Based on effectiveness and length, priority to most recent ones

Dual solutions from simplex iterations

- We can generate dual solutions at each dual simplex iteration
- Periodically check the $z(y), r_j(y)$ pair
- Keep the best pair that minimizes $\pi_j(y)$
- Mostly done anyway to check the objective cutoff



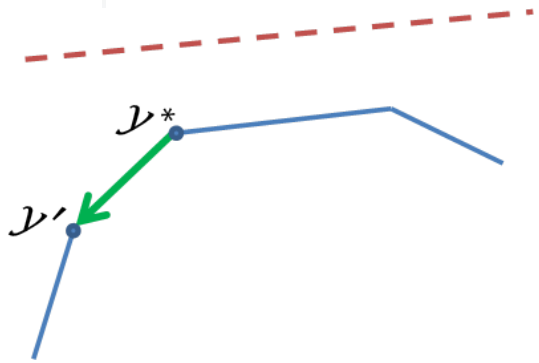
2. NEIGHBORHOOD SEARCH: OPTIMAL FACE



1. Set a basic, primal degenerate variable as leaving
2. Compute pivot row
3. Perform a simple dual ratio test to see how far the reduced cost can be pushed

- Objective value stays constant
- The new reduced cost is the dual steplength
- Nonbasic variables in the pivot row may also get an improved reduced cost
- Just a step on the dual optimal face

3. NEIGHBORHOOD SEARCH: NONOPTIMAL FACE

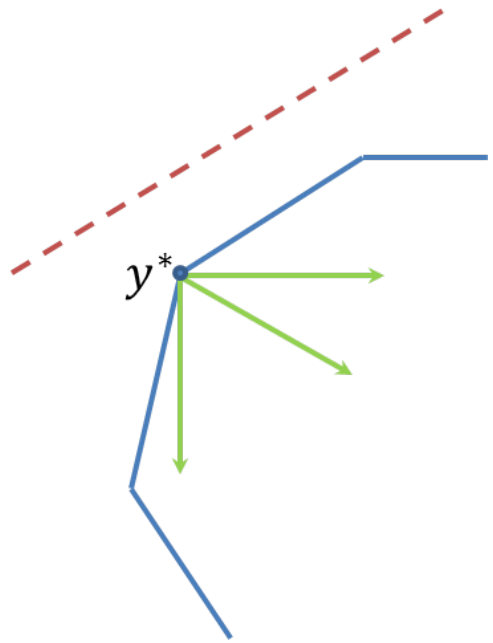


Extend to primal nondegenerate variables (incl. fractionals):

1. Set a basic, primal nondegenerate variable as leaving
   - The objective value will decrease
2. Compute pivot row
3. Perform a simple dual ratio test to see how far the reduced cost can be pushed
   - Pseudolinear, can check the derivative at the beginning
   - Hopefully the steplength is large enough, so that overall the tightening improves

This is still on the surface of the dual polyhedron, not inside.

DUAL TIGHTENING | 4. IMPROVING DIRECTION



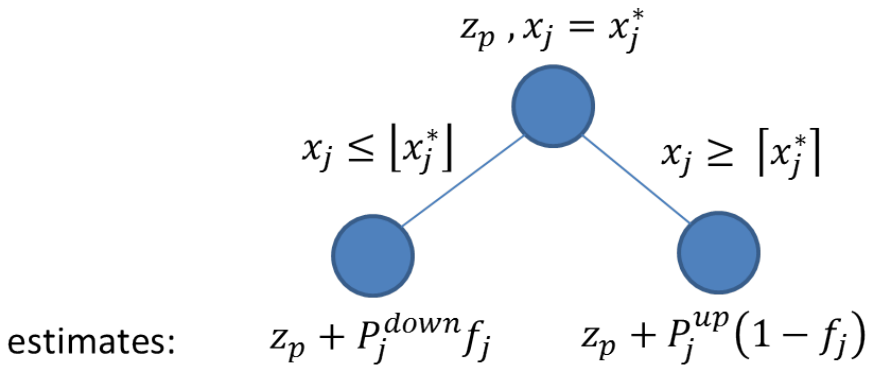
- Coordinate/gradient descent
- Move in the direction of a dual ray
- Move in the direction towards another dual solution

BRANCHING VARIABLE SELECTION

- Store dual solutions/rays and use them for variable selection
- However, we lose information as the dual information list changes dynamically
- Our solution:
  - Add information from dual solutions to pseudocosts
  - Add information from dual rays to conflict scores

TRADITIONAL PSEUDOCOSTS

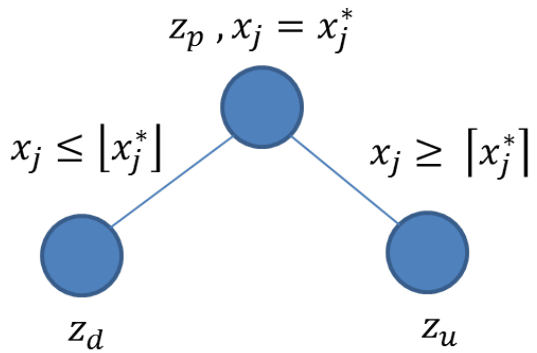
- The pseudocost of a variable is an estimate of change in the objective function if the variable is tightened by one unit
- P_j^{down} and P_j^{up}
- At a node t, let $f_j = x_j^* - \lfloor x_j^* \rfloor$.



- How do we maintain pseudocosts?

TRADITIONAL PSEUDOCOSTS

- Pseudocosts are updated after branching



- Objective change per unit at node t:

$$\delta_j^{\text{down}}(t) = \frac{z_d - z_p}{f_j} \quad , \quad \delta_j^{\text{up}}(t) = \frac{z_u - z_p}{1 - f_j}$$

- Pseudocost of variable j is the average:

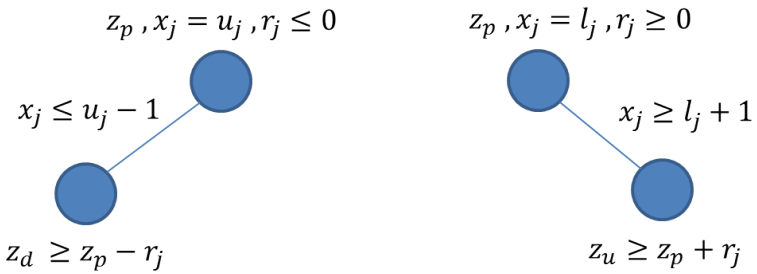
$$P_j^{\text{down}} = \frac{\sum_{t \in T_j} \delta_j^{\text{down}}(t)}{|T_j|} \quad , \quad P_j^{\text{up}} = \frac{\sum_{t \in T_j} \delta_j^{\text{up}}(t)}{|T_j|}$$

- They can be initialized/updated with strong branching

IMPROVED PSEUDOCOSTS

- Traditionally, pseudocosts are updated only by primal information
  - Only for the fractional/basic variable that is branched on
  - One update at a time
  - Only after the child is evaluated
- Improve pseudocosts using dual information
  - Through reduced costs of nonbasic variables
  - Whenever a new dual solution is found
  - Multiple updates

IMPROVED PSEUDOCOSTS



- Objective change (lower bound) per unit at node t:

$$\gamma_j^{\text{down}}(t) = -r_j \quad \text{if } r_j \leq 0 \quad , \quad \gamma_j^{\text{up}}(t) = r_j \quad \text{if } r_j \geq 0$$

- Improved pseudocost of variable j:

$$P_j^{\text{down}} = \frac{\sum_{t \in T_j} \delta_j^{\text{down}}(t) + \sum_{t \in S_j} \gamma_j^{\text{down}}(t)}{|T_j| + |S_j|}$$

$$P_j^{\text{up}} = \frac{\sum_{t \in T_j} \delta_j^{\text{up}}(t) + \sum_{t \in S_j} \gamma_j^{\text{up}}(t)}{|T_j| + |S_j|}$$

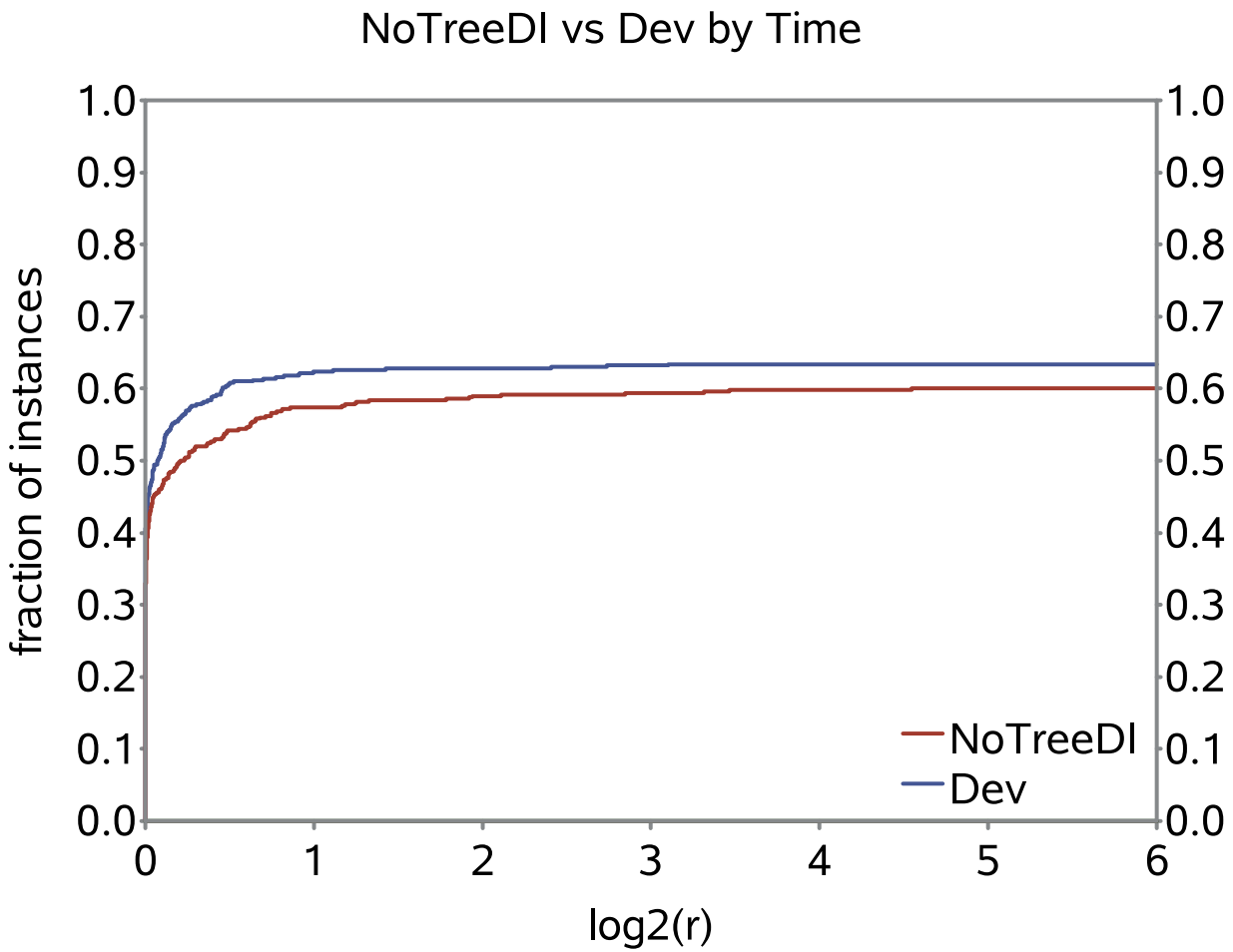
COMPUTATIONAL RESULTS

SAS/OR

- 500 test instances (internal + public)
- 1h time limit

COMPUTATIONAL RESULTS

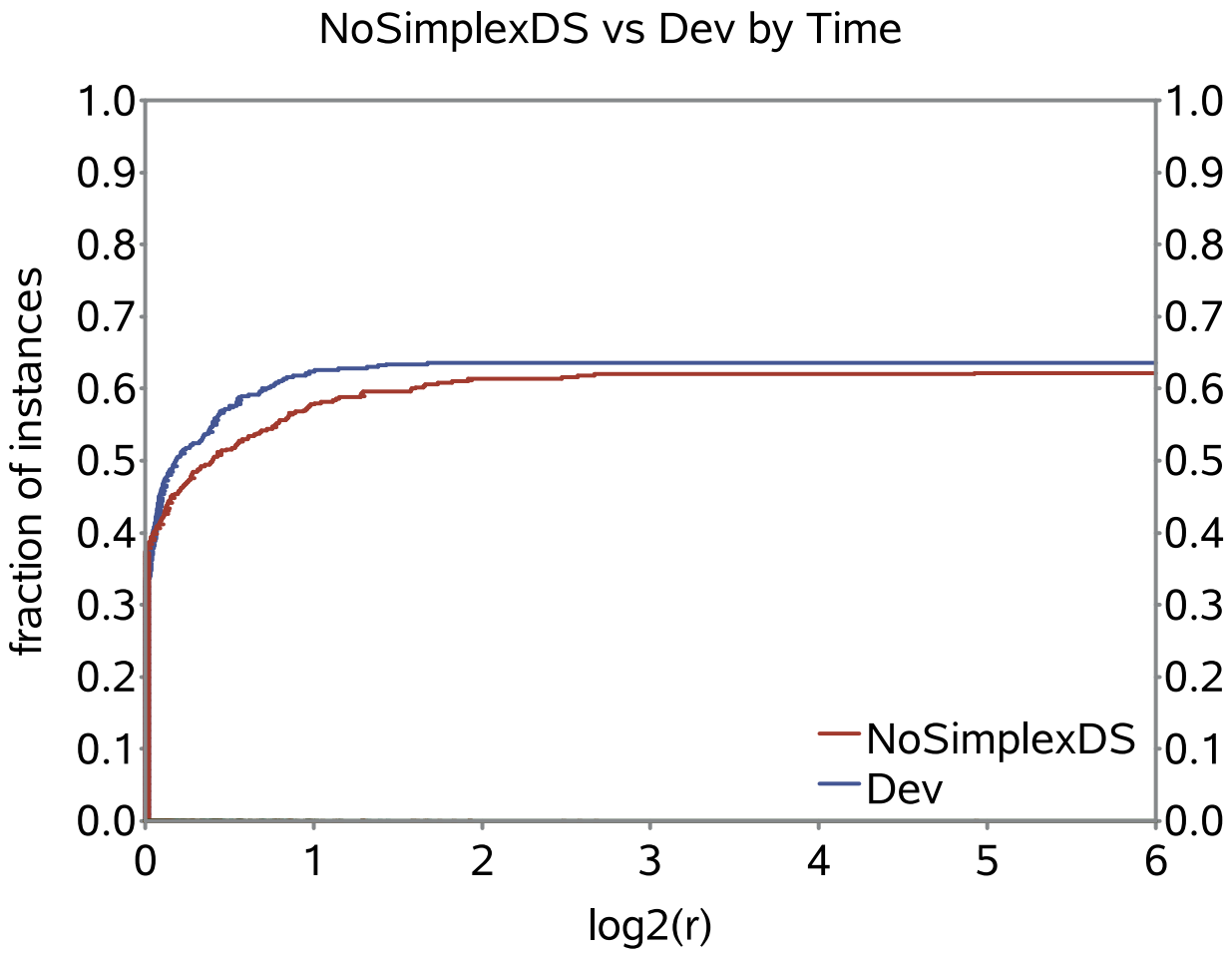
SAS/OR - DUAL INFORMATION FROM TREE



- 6% speedup + 17 instances

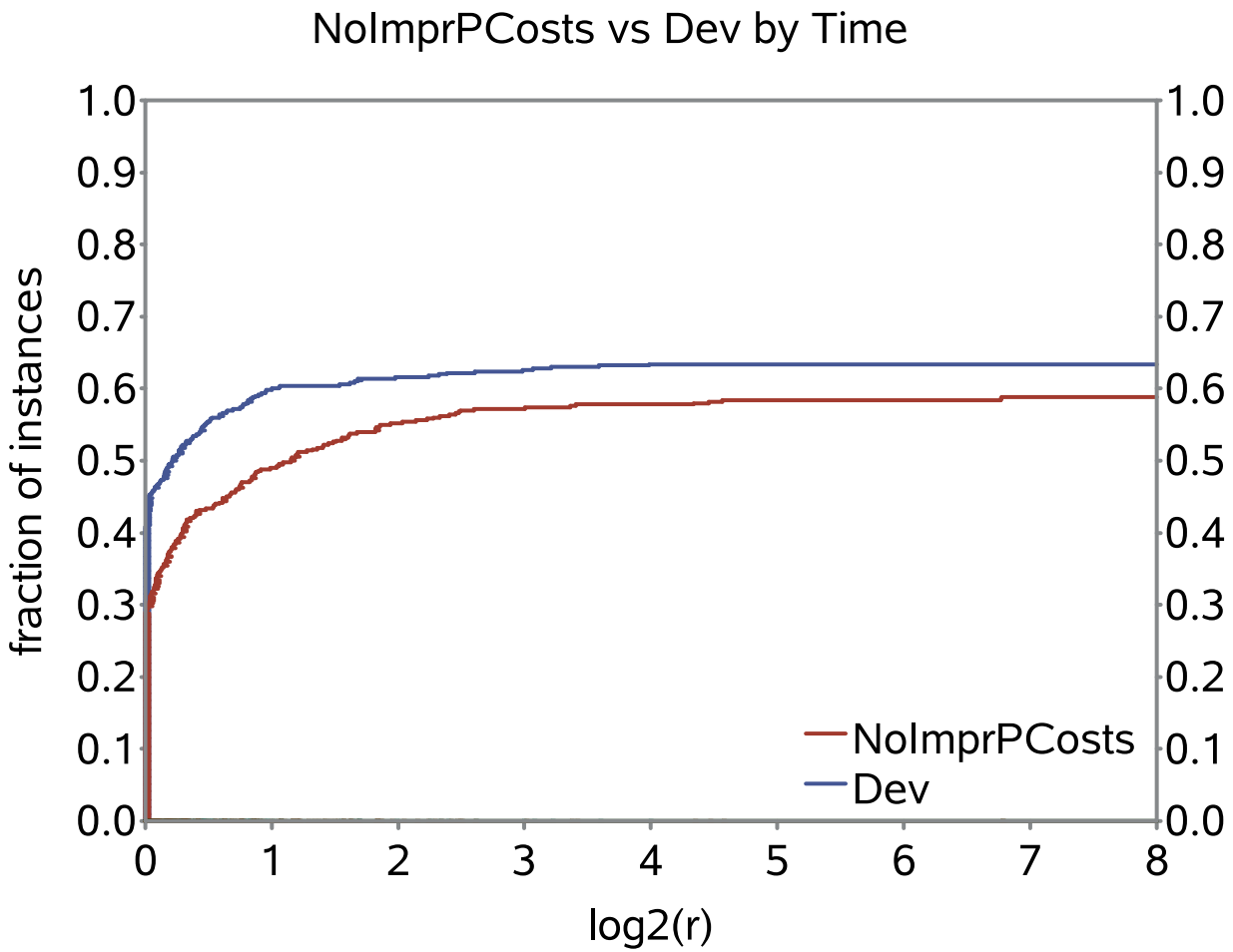
COMPUTATIONAL RESULTS

SAS/OR - DUAL SOLUTIONS FROM SIMPLEX



- 8% speedup + 7 instances

SAS/OR - IMPROVED PSEUDOCOSTS



- 18 % speedup + 23 instances

http://support.sas.com/or

More ways to use dual information in MILP