IE 496 Assignment #4 Due date: Tuesday, March 24, 4pm

Dr. Imre Pólik (34 points)

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1. (15 points) Consider the semidefinite programming problem in the standard format and assume that the constraint matrices $A_i, i = 1, \ldots, m$ all have rank one. Show how to streamline the computation of the Newton system with NT scaling. Recall that computational cost of forming the Newton system with fully dense data is $\mathcal{O}(mn^3 + m^2n^2)$. What is the cost with rank one constraint matrices?

2. (7 points) Prove that the Jordan product of two vectors from the second-order cone is in the second-order cone.

3. (12 points) Show that if the matrices

$$\begin{pmatrix} x_1 & x_4 \\ x_4 & x_2 \end{pmatrix} \text{ and } \begin{pmatrix} x_2 & x_5 \\ x_5 & x_3 \end{pmatrix}$$

are both positive semidefinite, then the value of t can be chosen to make

$$\begin{pmatrix} x_1 & x_4 & t \\ x_4 & x_2 & x_5 \\ t & x_5 & x_3 \end{pmatrix}$$

positive semidefinite. (Remark: This is important in decomposition methods. Imagine that in the coefficient matrices and the objective, the element corresponding to t is 0. In that case we can ignore that element, and replace the original positive semidefinite constraint with some smaller ones. After we get the optimal solution, we can reassemble the matrix and set the ignored values so that the matrix is positive semidefinite. More details to come in class.)