IE 496 Assignment #3 — Due date: Thursday, February 26, 4pm

Dr. Imre Pólik — (30 points)

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1. (7 points) Derive a general formula for the square root of a second-order vector. Discuss which vectors have a square root.

2. (8 points) Prove that for second-order vectors, although the multiplication $\circ$ is not associative, the trace operator is, i.e., $\operatorname{Tr}(u \circ (v \circ w)) = \operatorname{Tr}((u \circ v) \circ w)$, where $\operatorname{Tr}(x) = 2x_1$, where $x = (x_1, x_2, \ldots, x_n)$.

3. (15 points) Consider the following SDP:

$$
\begin{aligned}
\min \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} &\bullet X \\
\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} &\bullet X = 4 \\
\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} &\bullet X = 0 \\
X &\succeq 0
\end{aligned}
$$

   (a) Formulate the dual problem.

   (b) Formulate the equations for the central path.

   (c) Starting from the strictly feasible points

$$
\begin{aligned}
X &= \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix} \\
S &= \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix},
\end{aligned}
$$

   perform one iteration of the interior-point method described in class using NT scaling. Just find the solutions of the scaled Newton system, don't bother with the neighbourhoods and the stepsize. You can use some computer algebra system (Matlab or others) in the computations, i.e., you don't have to invert matrices manually.