IE 496 Assignment #2 — Due date: Tuesday, February 10, 4pm

Dr. Imre Pólik — (30 points)

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1. (10 points) Consider the following linear operator:

$$\mathcal{A} : \mathbb{R}^{n^2} \to \mathbb{R}^{n^2}$$
$$\mathcal{A}\mathrm{vec}(X) = \mathrm{vec}(UXU),$$

   where U is an $n \times n$ symmetric matrix and $\mathrm{vec}(X)$ is the vectorization of X, i.e., it contains the columns of X stacked together in one long column vector. Matrix $\mathcal{A}$ maps an $n \times n$ matrix to another one.

   (a) Form $\mathcal{A}$ explicitly in terms of U. Try with small instances first. What is the structure of this matrix? (Hint: $\otimes$)

   (b) Show that if $U \succeq 0$ then $\mathcal{A}$ is a positive semidefinite operator, meaning that $X \bullet (\mathcal{A}X) \geq 0$ for all X.

2. (10 points) Consider the SOCP problem

$$\begin{aligned} \min\ & c^T x \\ & Ax = b, \\ & x \in \mathbb{L}, \end{aligned}$$

   where

$$c = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, A = \begin{pmatrix} -1 & 0 & -1 \\ -1 & 0 & 1 \end{pmatrix}, b = \begin{pmatrix} -2 \\ 0 \end{pmatrix},$$

   and $\mathbb{L}$ is the second-order cone $\{(x0; x1; x2) : x_0 \geq \|(x_1; x_2)\|_2\}$.

   (a) Formulate the dual of this problem.

   (b) Show that, while the optimal values of these two problems are equal, one of these optimal values is not attained. Check if strict feasibility holds for the problems.

   (c) How does the situation change if b_2 is changed to -2ε for positive ε?

3. (10 points) Formulate the previous problem as an SDP in standard format. Write out the system defining the central path. Try to find the solution of the system as a function of μ. Does the central path exist? What is its limit point?