IE 496 Assignment #1 — Due date: Thursday, January 29, 4pm

Dr. Imre Pólik — (30 points)

Problems 1-3 are from Mike Todd.

1. (7 points) Suppose that instead of replacing the nonnegative vector x with a positive semidefinite matrix X in the standard-form LP, we replace each individual nonnegative variable by a positive semidefinite matrix. We might then get the problem (studied by Bellman and Fan in 1963!)

$$\begin{aligned} \min \sum_{j=1}^{n} C_j \bullet X_j & \\ \sum_{j=1}^{n} A_{ij} \bullet X_j &= b_i, \ \forall i = 1, \ldots, m \\ X_j &\succeq 0, \ \forall j = 1, \ldots, n. \end{aligned}$$

   Show how this can be formulated as an SDP.

2. (7 points) Suppose that X is positive definite. Show that $X - vv^T \succeq 0$ if and only if $v^T X^{-1} v \leq 1$.

3. (6 points) Show that the second-order constraint $\|Ay\|_2 \leq c^T x + d$ can be written using semidefinite constraints.

4. (10 points) Consider the following linear operator:

$$\begin{aligned} \mathcal{A} &: \mathbb{R}^{2\times 2} \to \mathbb{R}^2 \\ \mathcal{A}X &= AXe + XAe, \end{aligned}$$

   where A and X are 2×2 symmetric matrices and $e = (1, 1)$. Find the matrices A_1 and A_2 such that $\mathcal{A}X = (A_1 \bullet X, A_2 \bullet X)$. For some extra credit, generalize the result to the case when X and A are $n \times n$ matrices.