Preprocessing and stopping

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**What can we do?**

- Remove empty row?  
  
  Yes!

\[
\begin{align*}
\mathbf{A} & \begin{bmatrix} 
0 & \cdots & \cdots & \cdots & 0 
\end{bmatrix} = b_i \\
\mathbf{x} & = b_i \\
b_i & \begin{cases} 
= 0, \text{ remove row} \\
\neq 0, \text{ primal infeasible}
\end{cases}
\end{align*}
\]
What can we do?

- Remove empty row? Yes!
- Make $A$ full row rank? Yes! (Sparsity...)
What can we do?

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- Make $A$ full row rank? Yes! (Sparsity…)
- Remove singleton row (fixed variable)? No!
- Remove empty column (unconstrained variable)? No!
- Strengthen bounds, express variables? No!

Then what? Simpler, but equivalent forms!

$$
\begin{bmatrix}
0 & \cdots & 0 & a_{i,j} & 0 & \cdots & 0
\end{bmatrix}
= b_i
$$

$$
x_j = b_i / a_{i,j} \text{ and } x \succeq 0
$$

good if $x_j$ is diagonal and $b_i = 0$
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$x_j$ is unconstrained, but $x \succeq 0$

$s_j$ is fixed
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Bound and fixing constraints

- How to treat

\[ AX = b \]
\[ X_{ij} = a_{ij}, \text{ for some } i, j \]
\[ X \succeq 0 \]

or

\[ Ax = b \]
\[ x_i = a_i, \text{ for some } i, j \]
\[ x \in \mathbb{I} \]

- SOCP: aggregate the fixed variables
- Can be exploited to compute the Newton system faster
- SDP: No technique known
Decomposition techniques

- $S = C - A^*y \Rightarrow$ structure!
  - Block diagonal SDPs (Maybe linear?)
  - Narrow-band matrices
  - General completion scheme

- Second order formulation

$$
\begin{pmatrix}
  v_1 & v_2 & \ldots & v_n \\
  v_2 & v_1 & & \\
  \vdots & \ddots & \ddots & \\
  v_n & & v_1 \\
\end{pmatrix} \succeq 0 \iff (v_1, v_2:n) \in \mathcal{K}^q
$$

- Free variables
Self-dual embedding model for SDP

- Strictly interior starting point

\[
\begin{align*}
\min \ (x_0^T s_0 + 1) \theta \\
Ax & - b \tau + \bar{b} \theta & = 0 \\
-A^T y & + c \tau - \bar{c} \theta & = s \\
b^T y & - c^T x & + \bar{z} \theta & = \kappa \\
-\bar{b}^T y & - \bar{c}^T x & - \bar{z}^T \tau & = -x_0^T s_0 - 1
\end{align*}
\]

where

- \( x_0, s_0 \in \mathbb{R}^n, y_0 \in \mathbb{R}^m, \tau, \theta \in \mathbb{R} \)
- \( \bar{b} = b - Ax_0, \bar{c} = c - A^T y_0 - s_0, \bar{z} = c^T x_0 - b^T y_0 + 1 \)
- \( \tau > 0, \theta = 0: \ x/\tau, y/\tau, s/\tau \text{ are optimal} \)
- \( \tau = 0, \theta > 0: \text{ infeasibility} \)
- \( \tau = 0, \theta = 0: \ ? \)
Stopping criteria

- SD model with \((x^0, y^0, s^0)\)
- \(\tau \kappa \geq (1 - \beta) \mu\)
- Stop if
  \[
  \max \left\{ \|Ax - b\tau\|, \|ATy + s - c\tau\|, c^T x - b^T y \right\} \leq \varepsilon \tau
  \]
  \[
  \frac{1 - \beta}{1 + \rho} \geq \tau
  \]
- \(\varepsilon\)-optimum or \(x^0^T s^* + s^0^T x^* \geq \rho\)
- Stop if
  \[
  \max \left\{ \|Ax - b\tau\|, \|ATy + s - c\tau\|, c^T x - b^T y \right\} \leq \varepsilon \tau
  \]
  \[
  (\tau \|c\| + \theta \|\bar{c}\|) \bar{\rho} \leq b^T y
  \]
  \[
  (\tau \|b\| + \theta \|\bar{b}\|) \bar{\rho} \leq c^T x
  \]
- \(\varepsilon\)-optimum or \(\|\text{feasible}\| \geq \rho\)