

Preprocessing and stopping

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What can we do?

- Remove empty row?

Yes!

$$x$$

$$\begin{array}{c} A \\ 0 \dots \quad \dots \quad \dots 0 \end{array} = b_i$$

$$b_i \begin{cases} = 0, & \text{remove row} \\ \neq 0, & \text{primal infeasible} \end{cases}$$

What can we do?

- Remove empty row?
- Make \mathcal{A} full row rank?

Yes!

Yes! (Sparsity...)

Introduction

Basic questions

Bound and fixing
constraints

Decomposition

Stopping criteria

What can we do?

- Remove empty row? Yes!
- Make \mathcal{A} full row rank? Yes! (Sparsity...)
- Remove singleton row (fixed variable)? No!

$$x_j$$

$$\begin{bmatrix} 0 & \cdots & 0 & a_{i,j} & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} b_i \end{bmatrix}$$

$$x_j = b_i / a_{i,j} \text{ and } x \succeq 0$$

good if x_j is diagonal and $b_i = 0$

What can we do?

- Remove empty row? Yes!
- Make \mathcal{A} full row rank? Yes! (Sparsity...)
- Remove singleton row (fixed variable)? No!
- Remove empty column (unconstrained variable)? No!

$$\begin{array}{|c|} \hline x_j \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 0 \\ \hline \vdots \\ \hline 0 \\ \hline \end{array} = \begin{array}{|c|} \hline b \\ \hline \end{array}$$

x_j is unconstrained, but $x \succeq 0$
 s_j is fixed

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- Strengthen bounds, express variables? No!

What can we do?

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- Strengthen bounds, express variables? No!
- Then what? Simpler, but equivalent forms!

Bound and fixing constraints

- How to treat

$$\mathcal{A}X = b$$

$$X_{ij} = a_{ij}, \text{ for some } i, j$$

$$X \succeq 0$$

or

$$Ax = b$$

$$x_i = a_i, \text{ for some } i, j$$

$$x \in \mathbb{L}$$

- SOCP: aggregate the fixed variables
- Can be exploited to compute the Newton system faster
- SDP: No technique known

Decomposition techniques

- $S = C - \mathcal{A}^*y \Rightarrow$ structure!
 - Block diagonal SDPs (Maybe linear?)
 - Narrow-band matrices
 - General completion scheme
- Second order formulation

$$\begin{pmatrix} v_1 & v_2 & \dots & v_n \\ v_2 & v_1 & & \\ \vdots & & \ddots & \\ v_n & & & v_1 \end{pmatrix} \succeq 0 \iff (v_1, v_{2:n}) \in \mathcal{K}^q$$

- Free variables

Self-dual embedding model for SDP

- Strictly interior starting point

$$\begin{array}{rcll}
 & \min (x_0^T s_0 + 1)\theta & & \\
 Ax & -b\tau & +\bar{b}\theta & = 0 \\
 -A^T y & & +c\tau & -\bar{c}\theta & = s \\
 b^T y & -c^T x & & +\bar{z}\theta & = \kappa \\
 -\bar{b}^T y & -\bar{c}^T x & -\bar{z}^T \tau & & = -x_0^T s_0 - 1 \\
 & x \succeq 0, \tau \geq 0, s \succeq 0, \kappa \geq 0, & &
 \end{array}$$

where

- $x_0, s_0 \in \mathbb{R}^n, y_0 \in \mathbb{R}^m, \tau, \theta \in \mathbb{R}$
- $\bar{b} = b - Ax_0, \bar{c} = c - A^T y_0 - s_0, \bar{z} = c^T x_0 - b^T y_0 + 1$
- $\tau > 0, \theta = 0$: $x/\tau, y/\tau, s/\tau$ are optimal
- $\tau = 0, \theta > 0$: infeasibility
- $\tau = 0, \theta = 0$: ?

Stopping criteria

- SD model with (x^0, y^0, s^0)
- $\tau\kappa \geq (1 - \beta)\mu$
- Stop if

$$\max \{ \|Ax - b\tau\|^*, \|A^T y + s - c\tau\|, c^T x - b^T y \} \leq \varepsilon\tau$$

$$\frac{1 - \beta}{1 + \rho} \geq \tau$$

- ε -optimum or $x^{0T} s^* + s^{0T} x^* \geq \rho$
- Stop if

$$\max \{ \|Ax - b\tau\|^*, \|A^T y + s - c\tau\|, c^T x - b^T y \} \leq \varepsilon\tau$$

$$(\tau \|c\|^* + \theta \|\bar{c}\|^*) \bar{\rho} \leq b^T y$$

$$(\tau \|b\|^* + \theta \|\bar{b}\|^*) \bar{\rho} \leq c^T x$$

- ε -optimum or $\|\text{feasible}\| \geq \rho$