Preprocessing and stopping

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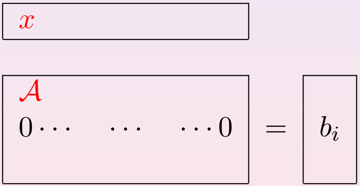
What can we do?

- Remove empty row? Yes!



$$b_i \begin{cases} = 0, & \text{remove row} \\ \neq 0, & \text{primal infeasible} \end{cases}$$

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$$x_j$$

$$\begin{bmatrix} 0\cdots0 & a_{i,j} & 0\cdots0 \end{bmatrix} = \begin{bmatrix} b_i \end{bmatrix}$$

$x_j = b_i/a_{i,j}$ and $x \succeq 0$

good if x_j is diagonal and $b_i = 0$

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- Make $\mathcal{A}$ full row rank? Yes! (Sparsity...)
- Remove singleton row (fixed variable)? No!
- Remove empty column (unconstrained variable)? No!

$$\begin{array}{c} \boxed{\quad x_j \quad} \\ \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \color{red}{b} \\ \\ \\ \end{bmatrix} \end{array}$$

x_j is unconstrained, but $x \succeq 0$

s_j is fixed

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- Strengthen bounds, express variables? No!

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- Then what? Simpler, but equivalent forms!

Bound and fixing constraints

- How to treat

$$\begin{aligned} \mathcal{A}X &= b \\ X_{ij} &= a_{ij}, \text{ for some } i,j \\ X &\succeq 0 \end{aligned}$$

or

$$\begin{aligned} Ax &= b \\ x_i &= a_i, \text{ for some } i,j \\ x &\in \mathbb{L} \end{aligned}$$

- SOCP: aggregate the fixed variables
- Can be exploited to compute the Newton system faster
- SDP: No technique known

Decomposition techniques

- $S = C - \mathcal{A}^* y \Rightarrow$ structure!
  - Block diagonal SDPs (Maybe linear?)
  - Narrow-band matrices
  - General completion scheme
- Second order formulation

$$\begin{pmatrix} v_1 & v_2 & \dots & v_n \\ v_2 & v_1 & & \\ \vdots & & \ddots & \\ v_n & & & v_1 \end{pmatrix} \succeq 0 \iff (v_1, v_{2:n}) \in \mathcal{K}^q$$

- Free variables

Self-dual embedding model for SDP

- Strictly interior starting point

$$
\begin{array}{rrrrcl}
\multicolumn{6}{c}{\min\ (x_0^T s_0 + 1)\theta} \\
 & Ax & -b\tau & +\bar{b}\theta & = & 0 \\
-A^T y & & +c\tau & -\bar{c}\theta & = & s \\
b^T y & -c^T x & & +\bar{z}\theta & = & \kappa \\
-\bar{b}^T y & -\bar{c}^T x & -\bar{z}^T\tau & & = & -x_0^T s_0 - 1 \\
\multicolumn{6}{c}{x \succeq 0,\ \tau \geq 0,\ s \succeq 0,\ \kappa \geq 0,}
\end{array}
$$

  where
  - $x_0, s_0 \in \mathbb{R}^n$, $y_0 \in \mathbb{R}^m$, $\tau, \theta \in \mathbb{R}$
  - $\bar{b} = b - Ax_0$, $\bar{c} = c - A^T y_0 - s_0$, $\bar{z} = c^T x_0 - b^T y_0 + 1$
- $\tau > 0$, $\theta = 0$: $x/\tau, y/\tau, s/\tau$ are optimal
- $\tau = 0$, $\theta > 0$: infeasibility
- $\tau = 0$, $\theta = 0$: ?

Stopping criteria

- SD model with (x^0, y^0, s^0)
- $\tau\kappa \geq (1-\beta)\mu$
- Stop if

$$\max\left\{\|Ax - b\tau\|^*, \left\|A^T y + s - c\tau\right\|, c^T x - b^T y\right\} \leq \varepsilon\tau$$
$$\frac{1-\beta}{1+\rho} \geq \tau$$

- ε-optimum or ${x^0}^T s^* + {s^0}^T x^* \geq \rho$
- Stop if

$$\max\left\{\|Ax - b\tau\|^*, \left\|A^T y + s - c\tau\right\|, c^T x - b^T y\right\} \leq \varepsilon\tau$$
$$\left(\tau \|c\|^* + \theta \|\bar{c}\|^*\right) \bar{\rho} \leq b^T y$$
$$\left(\tau \|b\|^* + \theta \left\|\bar{b}\right\|^*\right) \bar{\rho} \leq c^T x$$

- ε-optimum or $\|\text{feasible}\| \geq \rho$